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Design of Built-In Tests for Robust Active Fault Detection and Isolation of Discrete Faults in Uncertain Systems

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System Health Diagnostics in Safety-Critical Fields

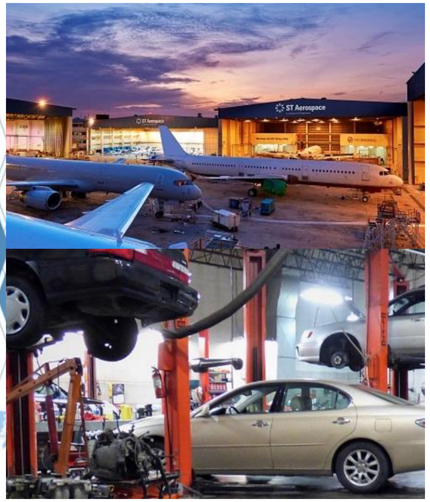
Improving fault diagnostics leads to increased test rigor and guarantees to safety

Goal of system health diagnostics

- High accuracy
- Quick resolutions

Tradeoff exists between test complexity and allotted time

- Cost
- Safety

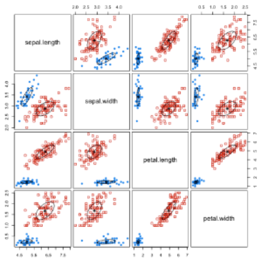


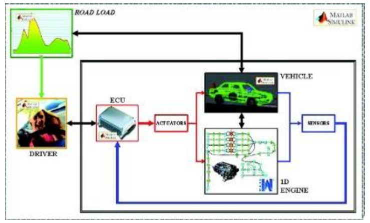
Active Model-Based Fault Detection and Isolation

Active model-based FDI methods provide accurate, low cost diagnostics

FDI methods

- Data-based
- Model-based





Passive FDI

- Analysis of system health during standard operation using measurements

Active FDI

- Incorporation of interruptive auxiliary input signals to improve system health analysis

- **Data-Based**
  - Qualitative Approach
    - Expert Systems
    - Frequency & Time-frequency Analysis
    - Fuzzy Logic
    - Pattern Recognition
    - Qualitative Trend Analysis
  - Quantitative Approach
    - Statistical
      - Principal Component Analysis
      - Partial Least Squares
      - Statistical Classifiers
    - Neural Network
- **Model-Based**
  - Qualitative Approach
    - Abstraction Hierarchy
      - Functional
      - Structural
    - Causal Model
      - Fault Trees
      - Qualitative Physics
      - Structural Graphs
  - Quantitative Approach
    - Parameter Estimation
      - Least Squares
      - Recursive Least Squares
      - Regression Analysis
    - Parity Space
      - Input-Output
      - State-Space
    - State Estimation
      - Kalman Filter
      - Observer
    - State & Parameter Estimation
      - Extended Kalman Filter
      - Two-Stage Kalman Filter

Impact of Uncertainty on Maintenance

Costly no fault found, false alarm, and non-detection events occur frequently during maintenance

Uncertainty negatively impacts system health diagnostics

- Major cause of false alarms and no fault founds
- Increases cost and maintenance time and decreases safety

No fault found events (NFFs) increase the potential of inserting a faulty system back into operation

- 30-50% of LRUs removed for maintenance in the aerospace industry are tagged as NFF [1]–[3]
- Over 90% of aircraft electronics maintenance costs can be attributed to NFFs [4]

<u>Problem:</u> The absence of faults due to uncertainty during ill-designed maintenance tests is a main cause for NFFs [2], [5]

[1] P. Söderholm, "A system view of the no fault found (NFF) phenomenon," Reliab. Eng. Syst. Saf., vol. 92, pp. 1–14, Jan. 2007.
[2] S. Khan, P. Phillips, I. Jennions, and C. Hockley, "No fault found events in maintenance engineering part 1: Current trends, implications and organizational practices," Reliab. Eng. Syst. Saf., vol. 123, pp. 183–195, Mar. 2014.
[3] I. James, D. Lumbard, I. Willis, and J. Goble, "Investigating no fault found in the aerospace industry," in Proceedings of Annual Reliability and Maintainability Symposium (Cat. No.03CH37415), pp. 441–446, IEEE, Jan. 2003.
[4] R. Williams, J. Banner, I. Knowles, M. Dube, M. Natishan, and M. Pecht, "An investigation of 'cannot duplicate' failures," Qual. Reliab. Eng. Int., vol. 14, pp. 331–337, Sep. 1998.
[5] S. Khan, P. Phillips, C. Hockley, and I. Jennions, "No fault found events in maintenance engineering part 2: Root causes, technical developments and future research," Reliab. Eng. Syst. Saf., vol. 123, pp. 196–208, Mar. 2014.

Global Optimization for Built-In Test Input Design

For safety-critical systems, the conservative approach for BIT design suffices

Goal: Develop a maintenance test (Built-In Test (BIT)) that produces unique system responses for a fault-free system and all of its fault scenarios even at its worst-case scenario of uncertainty

Method: Utilize global optimization techniques to solve a max-min program, reformulated as a semi-infinite program, involving the system inputs and uncertainty

The max-min approach is often considered to be sub-optimal due to its conservative nature of "raising the floor", i.e. finding the *best* worst-case

However, for safety-critical systems with strict regulations such as in the aerospace industry, this approach is sufficient due to its guarantees

Mathematical Formulation

Model, output, max-min, and implicit function equations

Model equations: $\mathbf{f}^{[f]}\left(\tilde{\mathbf{x}},\mathbf{u},\theta_p,\theta_u,\theta_f\right)=\mathbf{0},\quad \forall f\in\{0,1,\ldots,N_f\}\qquad (1)$

Output equations: $\mathbf{y}^{[f]}=\bar{\mathbf{x}}+\mathbf{w}\qquad (2)$

Max-min program:

$$\max_{\mathbf{u}\in U}\ \min_{\theta_u\in\Theta_u,\theta_f\in\Theta_f}\ G(\tilde{\mathbf{x}},\mathbf{u},\theta_p,\theta_u,\theta_f)\qquad (3)$$

$$\text{s.t. }\mathbf{f}(\tilde{\mathbf{x}},\mathbf{u},\theta_p,\theta_u,\theta_f)=(\mathbf{f}^{[1]},\mathbf{f}^{[2]},\ldots,\mathbf{f}^{[N_f]})=\mathbf{0}$$

Implicit function: $\tilde{\mathbf{x}}=\mathbf{x}(\mathbf{u},\theta_p,\theta_u,\theta_f)$ **exists such that**

$$\mathbf{f}(\mathbf{x}(\mathbf{u},\theta_p,\theta_u,\theta_f),\mathbf{u},\theta_p,\theta_u,\theta_f)=0,\quad\forall(\mathbf{u},\theta_p,\theta_u,\theta_f)\in U\times\Theta_p\times\Theta_u\times\Theta_f$$

is satisfied

$$\begin{aligned}
[f] &: \text{ fault scenario}\\
\mathbf{f}^{[f]} &: \text{ governing equations}\\
\tilde{\mathbf{x}}\in\tilde{X}\subset\mathbb{R}^{N_x} &: \text{ system states}\\
\mathbf{u}\in U\subset\mathbb{R}^{N_u} &: \text{ inputs}\\
\theta_p\in\Theta_p\subset\mathbb{R}^{N_{\theta_p}} &: \text{ design parameters}\\
\theta_u\in\Theta_u\subset\mathbb{R}^{N_{\theta_u}} &: \text{ uncertain parameters}\\
\theta_f\in\Theta_f\subset\mathbb{R}^{N_{\theta_f}} &: \text{ fault parameters}\\
\mathbf{y}^{[f]}\in Y\subset\mathbb{R}^{N_y} &: \text{ outputs}\\
\bar{\mathbf{x}}\in\bar{X}\subseteq\tilde{X}\subset\mathbb{R}^{N_y} &: \text{ measured states}\\
\mathbf{w}\in W\subset\mathbb{R}^{N_y} &: \text{ noise}\\
\mathbf{f} &: \text{ combined equations}\\
G &: \text{ objective function}\\
\mathbf{x} &: \text{ implicit function}
\end{aligned}$$

Extensive semi-infinite program (SIP):

$$\min_{\mathbf{u}\in U,\ \eta\in H} \ -\eta$$

$$\text{s.t. } g(\mathbf{x}(\mathbf{u},\theta_p,\theta_u,\theta_f),\mathbf{u},\theta_p,\theta_u,\theta_f,\eta) = \eta - G(\mathbf{x}(\mathbf{u},\theta_p,\theta_u,\theta_f),\mathbf{u},\theta_p,\theta_u,\theta_f) \leq 0 \quad (4)$$

$$\forall\ (\theta_u,\theta_f) \in \Theta_u \times \Theta_f$$

Feasibility criterion:

$$g(\mathbf{x}(\mathbf{u},\theta_p,\theta_u,\theta_f),\mathbf{u},\theta_p,\theta_u,\theta_f,\eta) = \eta + \eta_{feas} - \sum_{i=1}^{N_y}\sum_{f=0}^{N_f-1}\sum_{g=f+1}^{N_f}(y_i^{[f]} - y_i^{[g]})^2 \quad (5)$$

Worst-case BIT design (WCD) SIP:

$$\min_{\mathbf{u}\in U,\ \eta\in H} \ -\eta$$

$$\text{s.t. } \eta + \eta_{feas} - \sum_{i=1}^{N_y}\sum_{f=0}^{N_f-1}\sum_{g=f+1}^{N_f}(y_i^{[f]} - y_i^{[g]})^2 \leq 0 \quad (6)$$

$$\forall\ (\theta_u,\theta_f) \in \Theta_u \times \Theta_f$$

$\eta \in H \subset \mathbb{R}$:	SIP auxiliary variable
g :	Feasibility criterion
η_{feas} :	BIT feasibility parameter
N_f :	number of faults
N_y :	number of outputs
$y_i^{[f],[g]}$:	i^{th} output of fault scenarios f, g

Worst-Case Design Algorithms

Blankenship and Falk cutting plane and Mitsos right-hand side restriction algorithms used

Worst-case BIT design algorithm [6]–[8]

- Initialize uncertainty
- Set iteration count to 1
- Begin iteration
  - Solve outer program for BIT design, analyzing all previous uncertainty sets
  - Solve inner program at BIT design, for updated worst-case uncertainty set
  - Update iteration count
- Examine continuation criteria
  - If true, begin next iteration
  - If false, end algorithm, worst-case design found

Algorithm 1 SIP Max-Min Algorithm

Require: $\theta_u^{[1]} \in \Theta_u, \theta_f^{[1]} \in \Theta_f$

1: $K \leftarrow 1$

2: **while** $\hat{\eta}^{[K]} < \eta^{[K]} \wedge K \leq K_{max}$ **do**

3: $(\eta^{[K]}, \mathbf{u}^{[K]}) \leftarrow \min_{\mathbf{u} \in U,\ \eta \in H} -\eta$

s.t. $\eta - G(\mathbf{x}(\mathbf{u}, \theta_p, \theta_u^{[k]}, \theta_f^{[k]}), \mathbf{u}, \theta_p, \theta_u^{[k]}, \theta_f^{[k]}) \leq 0,$

$\forall k \in \{1, 2, \ldots, K\}$

4: $(\hat{\eta}^{[K]}, \theta_u^{[K+1]}, \theta_f^{[K+1]}) \leftarrow$

$\min_{\theta_u \in \Theta_u, \theta_f \in \Theta_f} G(\mathbf{x}(\mathbf{u}^{[K]}, \theta_p, \theta_u, \theta_f), \mathbf{u}^{[K]}, \theta_p, \theta_u, \theta_f)$

5: $K \leftarrow K + 1$

6: **end**

7: $(\mathbf{u}^{opt}, \theta_u^{opt}, \theta_f^{opt}) \leftarrow (\mathbf{u}^{[K-1]}, \theta_u^{[K]}, \theta_f^{[K]})$

julia & EAGO [9]

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[6] S. P. Asprey and S. Macchietto (2002). Designing robust optimal dynamic experiments. *Journal of Process Control*, *12*(4), 545–556.
[7] J. W. Blankenship and J. E. Falk, Infinitely constrained optimization problems, *Journal of Optimization Theory and Applications*, vol. 19, no. 2, pp. 261-281, 1976.
[8] Mitsos, A. Global Optimization of Semi-Infinite Programs via Restriction of the Right-Hand Side. *Optimization*. 60:10-1,1291-1308, 2011
[9] Wilhelm, Matthew; Stuber, Matthew (October 2017) Easy Advanced Global Optimization (EAGO): An Open-Source Platform for Robust and Global Optimization in Julia. Presented at the AIChE Annual Meeting in Minneapolis, MN.

Case Study: Three Tank System Description

Three tank system is a benchmark for FDI

Inputs: $\mathbf{u} \in \mathbf{U} = \left[10^{-5}, 10^{-4}\right]^2$
Pump 1 - Flow Rate u_1
Pump 2 - Flow Rate u_2

Outputs: $\mathbf{y} \in \mathbf{Y}$
Tank 1 - Level $\tilde{x}_1$
Tank 2 - Level $\tilde{x}_2$
Tank 3 - Level $\tilde{x}_3$

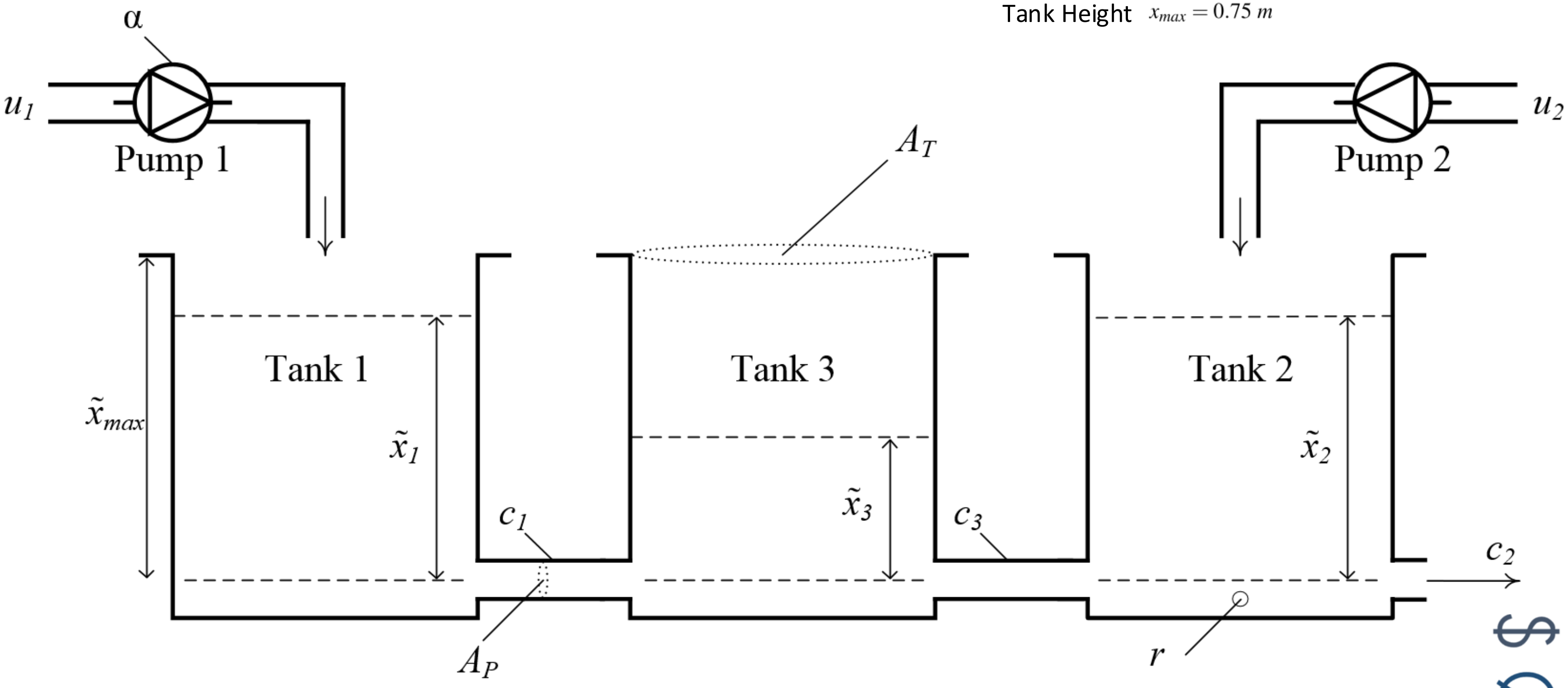
Faults Studied: $\theta_f \in \Theta_f$
Pump 1 - Failure $\alpha \sim \mathcal{N}(0.6, 0.0004)$
Tank 2 - Leak $r \sim \mathcal{N}(0.002, 0.000001)$

Uncertainties Present: $\theta_u \in \Theta_u$
Valve 13 - Outflow Coefficient $c_1 \sim \mathcal{N}(1, 0.0025)$
Valve 32 - Outflow Coefficient $c_2 \sim \mathcal{N}(0.8, 0.0025)$
Valve 20 - Outflow Coefficient $c_3 \sim \mathcal{N}(1, 0.0025)$

Design/Model Parameters: $\theta_p \in \Theta_p$
Tank Cross-Sec Area $A_T = 0.0154\ m^2$
Pipe Cross-Sec Area $A_P = 0.00005\ m^2$
Signum Approximation Constant $k = 1000$
Absolute Value Approximation Constant $\epsilon = 0.00001$
Gravitational Acceleration Constant $g = 9.80655\ m\,s^{-2}$
Pi $\pi = 3.14159265358979$
Tank Height $x_{max} = 0.75\ m$

$$\mathbf{f}(\tilde{\mathbf{x}}, \mathbf{u}, \theta_p, \theta_u, \theta_f) = (\mathbf{f}^{[0]}, \mathbf{f}^{[1]}, \mathbf{f}^{[2]}, \mathbf{f}^{[3]}) = \mathbf{0} = \qquad {}^*\Delta\tilde{x}_{ij}^{[f]} \equiv \tilde{x}_i^{[f]} - \tilde{x}_j^{[f]}$$

$$\begin{pmatrix}
-c_1 A_P \tanh(k\Delta\tilde{x}_{13}^{[0]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{13}^{[0]})^2+\epsilon}} + u_1 \\
-c_3 A_P \tanh(k\Delta\tilde{x}_{23}^{[0]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{23}^{[0]})^2+\epsilon}} - c_2 A_P\sqrt{2g\tilde{x}_2^{[0]}} + u_2 \\
c_1 A_P \tanh(k\Delta\tilde{x}_{13}^{[0]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{13}^{[0]})^2+\epsilon}} - c_3 A_P \tanh(k\Delta\tilde{x}_{32}^{[0]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{32}^{[0]})^2+\epsilon}} \\
-c_1 A_P \tanh(k\Delta\tilde{x}_{13}^{[1]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{13}^{[1]})^2+\epsilon}} + u_1 + (\alpha_1 - 1)u_1 \\
-c_3 A_P \tanh(k\Delta\tilde{x}_{23}^{[1]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{23}^{[1]})^2+\epsilon}} - c_2 A_P\sqrt{2g\tilde{x}_2^{[1]}} + u_2 \\
c_1 A_P \tanh(k\Delta\tilde{x}_{13}^{[1]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{13}^{[1]})^2+\epsilon}} - c_3 A_P \tanh(k\Delta\tilde{x}_{32}^{[1]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{32}^{[1]})^2+\epsilon}} \\
-c_1 A_P \tanh(k\Delta\tilde{x}_{13}^{[2]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{13}^{[2]})^2+\epsilon}} + u_1 \\
-c_3 A_P \tanh(k\Delta\tilde{x}_{23}^{[2]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{23}^{[2]})^2+\epsilon}} - c_2 A_P\sqrt{2g\tilde{x}_2^{[2]}} - c_2\pi r_L^2\sqrt{2g\tilde{x}_2^{[2]}} + u_2 \\
c_1 A_P \tanh(k\Delta\tilde{x}_{13}^{[2]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{13}^{[2]})^2+\epsilon}} - c_3 A_P \tanh(k\Delta\tilde{x}_{32}^{[2]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{32}^{[2]})^2+\epsilon}} \\
-c_1 A_P \tanh(k\Delta\tilde{x}_{13}^{[3]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{13}^{[3]})^2+\epsilon}} + u_1 + (\alpha_1 - 1)u_1 \\
-c_3 A_P \tanh(k\Delta\tilde{x}_{23}^{[3]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{23}^{[3]})^2+\epsilon}} - c_2 A_P\sqrt{2g\tilde{x}_2^{[3]}} - c_2\pi r_L^2\sqrt{2g\tilde{x}_2^{[3]}} + u_2 \\
c_1 A_P \tanh(k\Delta\tilde{x}_{13}^{[3]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{13}^{[3]})^2+\epsilon}} - c_3 A_P \tanh(k\Delta\tilde{x}_{32}^{[3]})\sqrt{2g\sqrt{(\Delta\tilde{x}_{32}^{[3]})^2+\epsilon}}
\end{pmatrix}$$



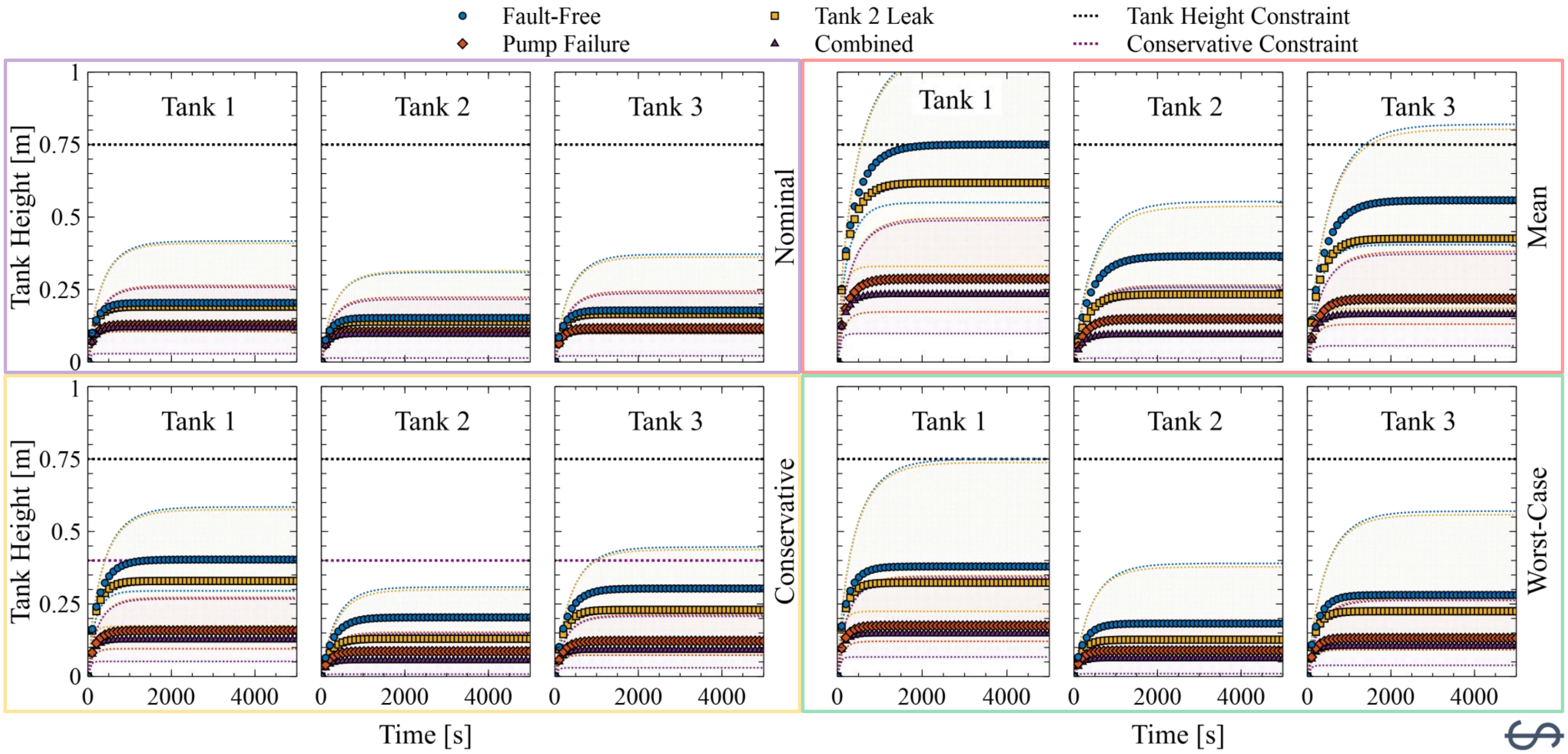
Case Study: Different BIT Designs

Four different operating conditions were analyzed for BIT effectiveness

BIT designs

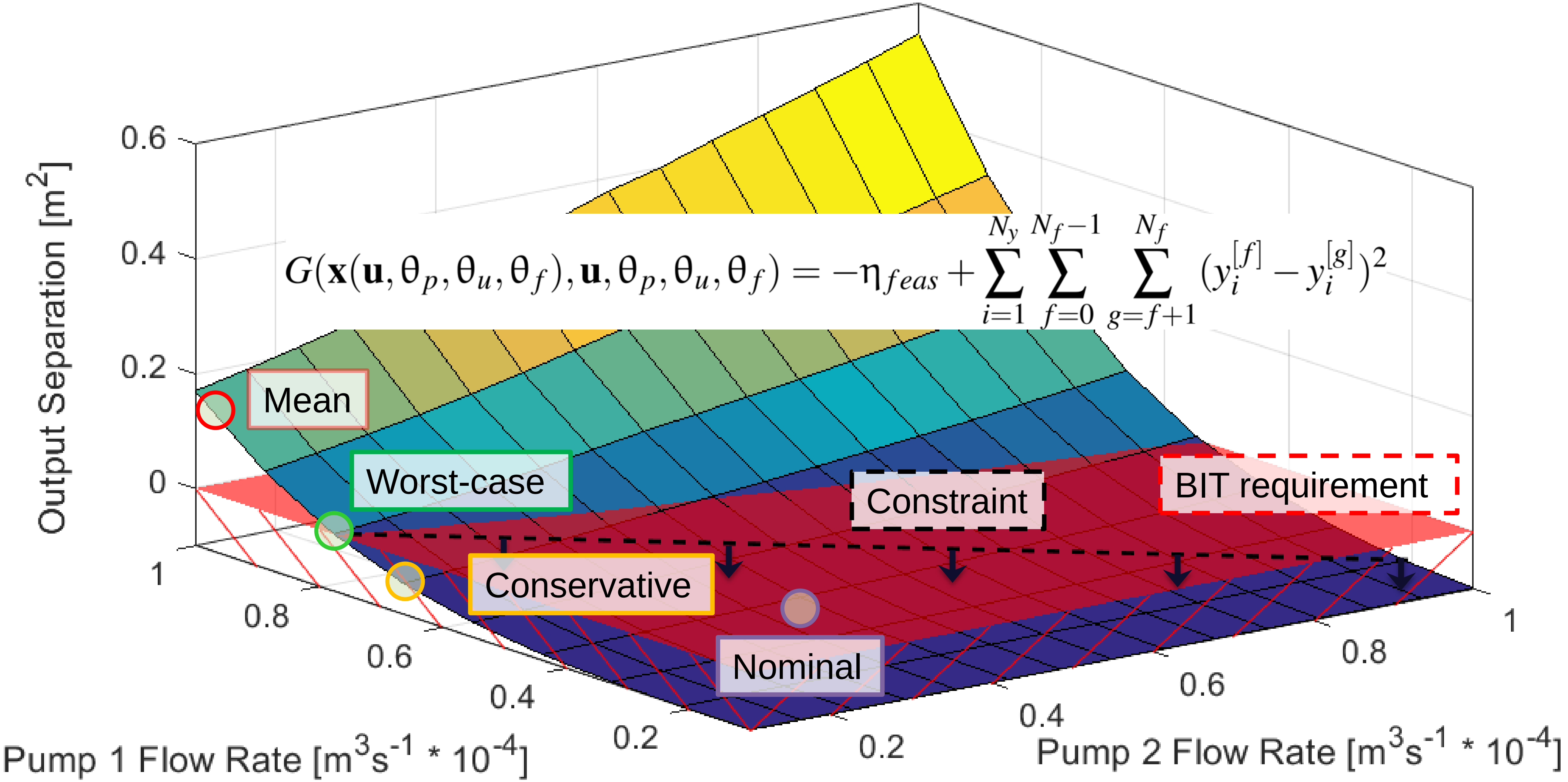
- ☐ Nominal
- ☐ Mean
- ☐ Conservative
- ☐ Worst-case

Inputs ($m^3 s^{-1} * 10^{-4}$)	u_1	u_2
Minimum	0.10	0.10
Nominal	0.41	0.41
Mean	0.97	0.10
Conservative	0.70	0.10
Worst-case	0.80	0.10
Maximum	1.00	1.00



Case Study: Objective Function Surface

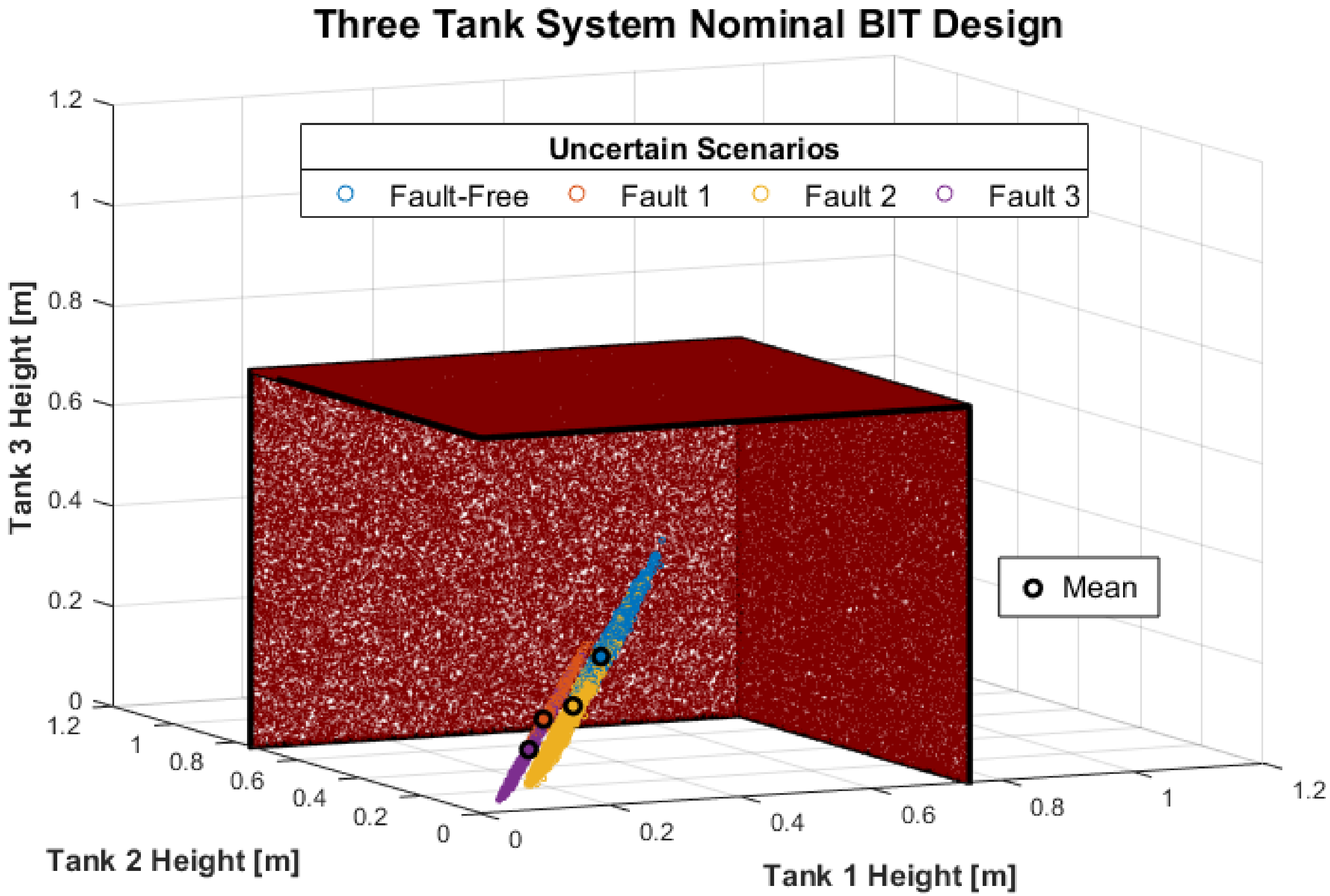
WCD lies on the intersection of the tank height constraint and the objective function G



$$G(\mathbf{x}(\mathbf{u},\theta_p,\theta_u,\theta_f),\mathbf{u},\theta_p,\theta_u,\theta_f) = -\eta_{feas} + \sum_{i=1}^{N_y} \sum_{f=0}^{N_f-1} \sum_{g=f+1}^{N_f} (y_i^{[f]} - y_i^{[g]})^2$$

Case Study: Nominal BIT Design

Poor separation of anticipated outputs and distribution overlap



Separation of anticipated outputs:

$$G(\mathbf{x}(\mathbf{u},\theta_p,\theta_u,\theta_f),\mathbf{u},\theta_p,\theta_u,\theta_f) =$$

$$-\eta_{feas} + \sum_{i=1}^{N_y} \sum_{f=0}^{N_f-1} \sum_{g=f+1}^{N_f} (\bar{y}_i^{[f]} - \bar{y}_i^{[g]})^2$$

$$= -0.1 + 0.14 = \boxed{0.04}$$

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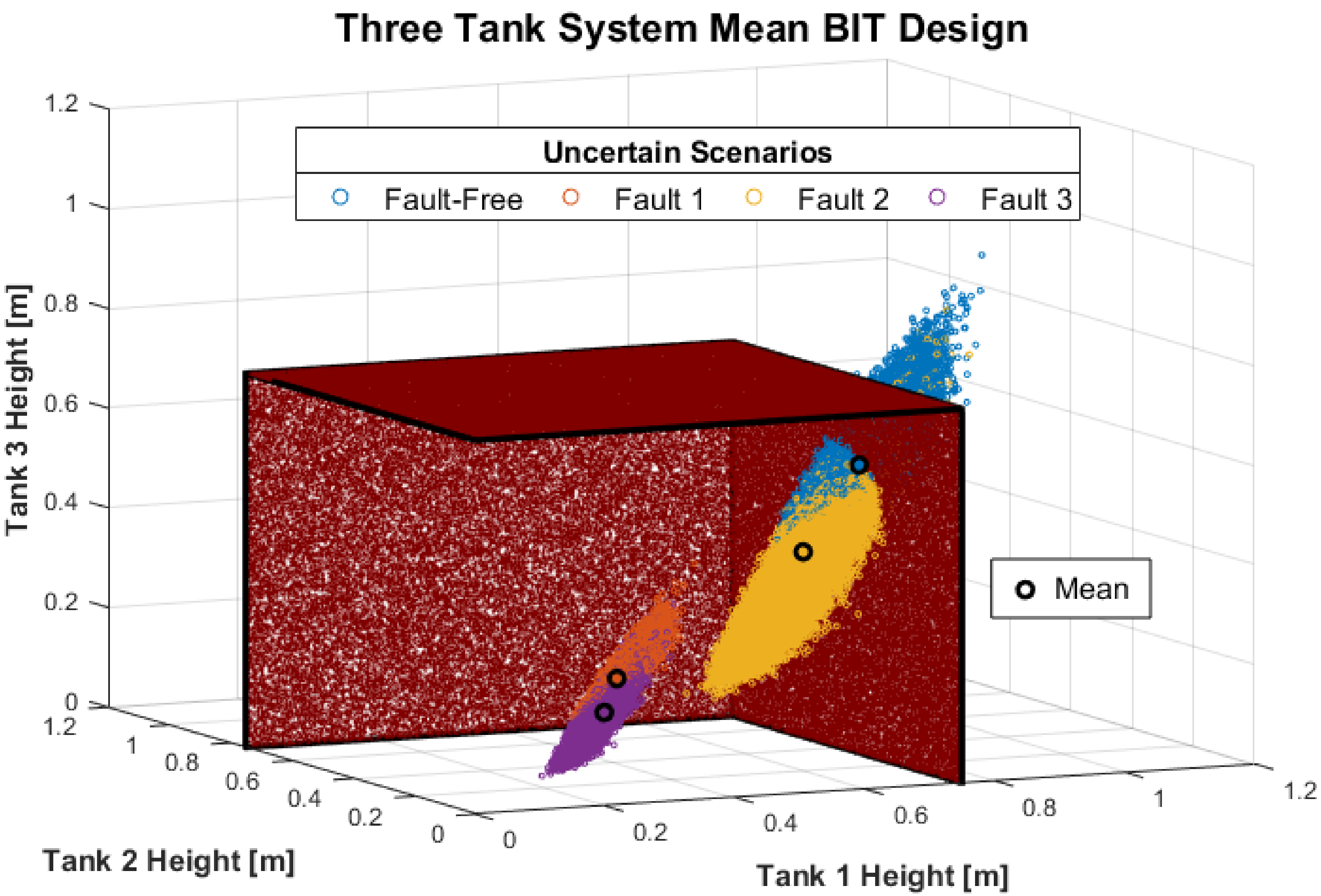
Case Study: Mean BIT Design

Improved separation, but violates constraints for numerous cases of uncertainty



Separation of anticipated outputs:

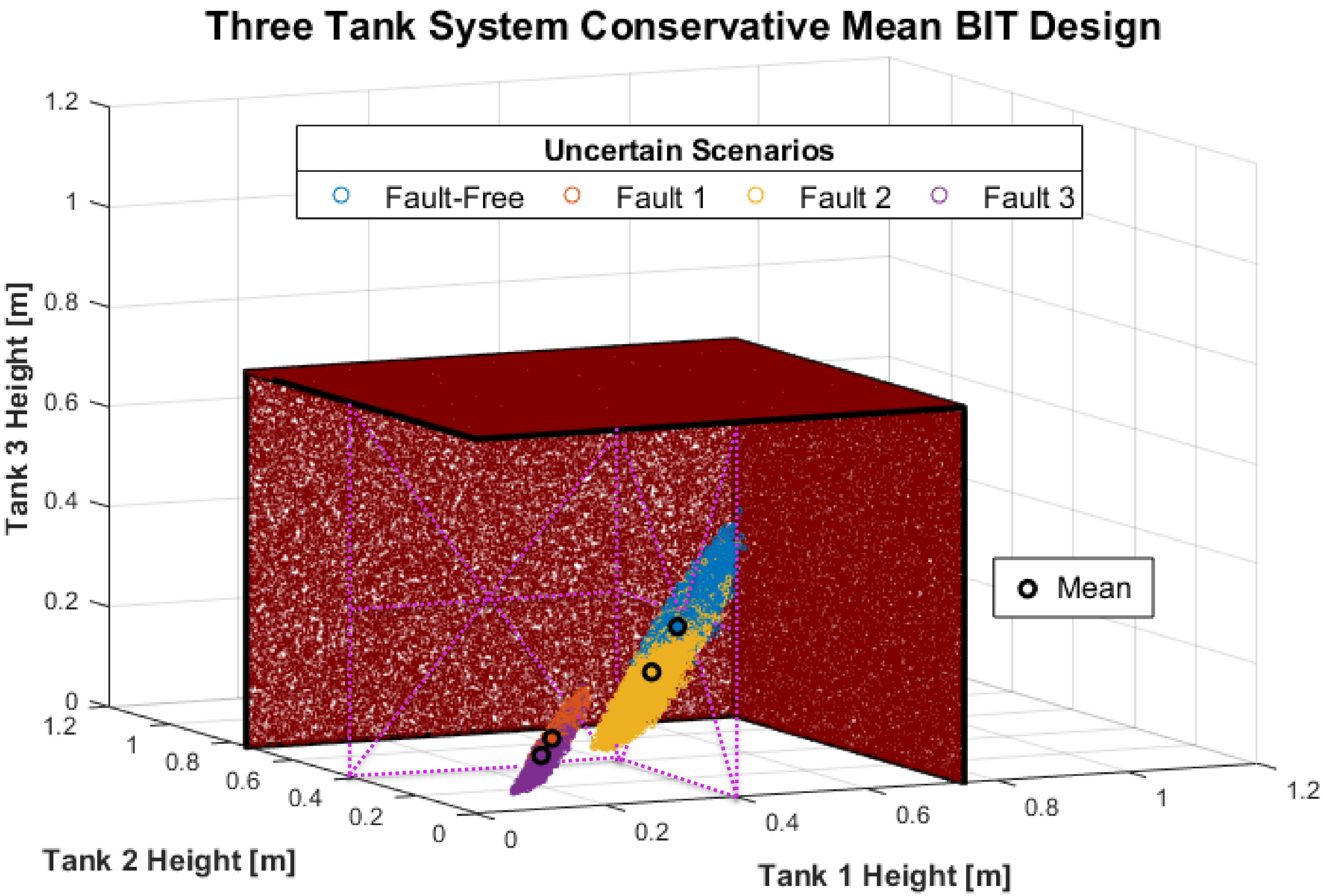
$$G(\mathbf{x}(\mathbf{u},\theta_p,\theta_u,\theta_f),\mathbf{u},\theta_p,\theta_u,\theta_f) =$$

$$-\eta_{feas} + \sum_{i=1}^{N_y}\sum_{f=0}^{N_f-1}\sum_{g=f+1}^{N_f}(\bar{y}_i^{[f]} - \bar{y}_i^{[g]})^2$$

$$= -0.1 + 1.33 = \boxed{1.23}$$

Case Study: Mean BIT Design w/ Conservative Constraint

Manages tank height constraint violations but results in underperformance of separation



Separation of anticipated outputs:

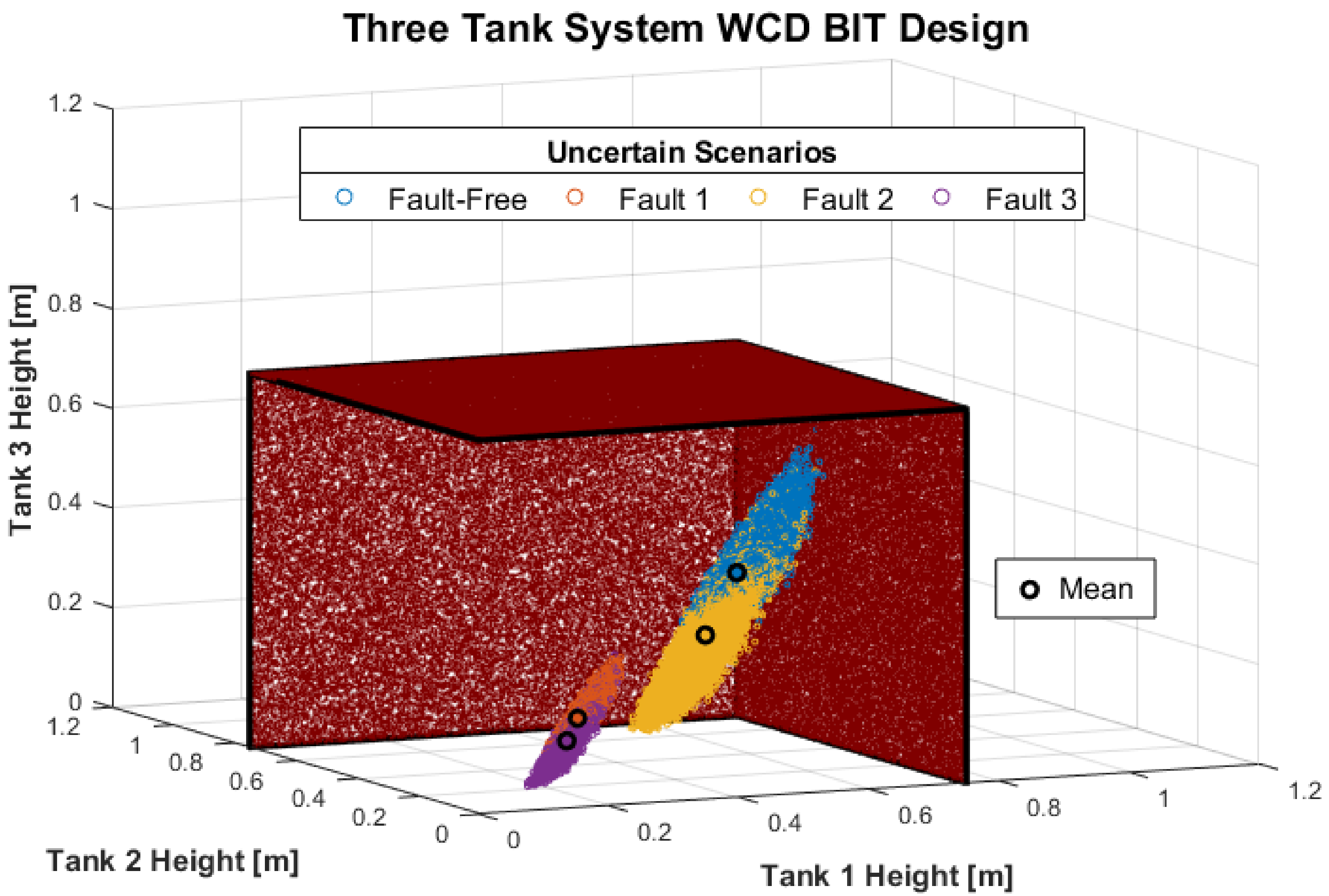
$$G(\mathbf{x}(\mathbf{u},\theta_p,\theta_u,\theta_f),\mathbf{u},\theta_p,\theta_u,\theta_f) =$$

$$-\eta_{feas} + \sum_{i=1}^{N_y}\sum_{f=0}^{N_f-1}\sum_{g=f+1}^{N_f}(\bar{y}_i^{[f]} - \bar{y}_i^{[g]})^2$$

$$= -0.1 + 0.38 = \boxed{0.28}$$

Case Study: Worst-Case BIT Design

Maximizes separation and maintains constraint feasibility for all uncertainty scenarios



Separation of anticipated outputs:

$$G(\mathbf{x}(\mathbf{u},\theta_p,\theta_u,\theta_f),\mathbf{u},\theta_p,\theta_u,\theta_f) =$$

$$-\eta_{feas} + \sum_{i=1}^{N_y}\sum_{f=0}^{N_f-1}\sum_{g=f+1}^{N_f}(\bar{y}_i^{[f]} - \bar{y}_i^{[g]})^2$$

$$= -0.1 + 0.63 = \boxed{0.53}$$

Conclusions

Method developed aims at improving fault detection and isolation at the worst-case scenario(s) of uncertainty

BIT design at the worst-case scenario of uncertainty shows improvement in output separation in comparison to the nominal, mean, and conservative mean BIT designs

Global feasibility provided, guaranteeing robustness of the BIT design which is important for safety-critical systems

Acknowledgements

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EAGO

[9] Wilhelm, Matthew; Stuber, Matthew (October 2017) Easy Advanced Global Optimization (EAGO): An Open-Source Platform for Robust and Global Optimization in Julia. Presented at the AIChE Annual Meeting in Minneapolis, MN.

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