UCONN
UNIVERSITY OF CONNECTICUT

Optimal Design of Controlled Environment Agricultural Systems Under Market Uncertainty

Shaylin A. Cetegen and Matthew D. Stuber*

PhD Student, Massachusetts Institute of Technology

Assistant Professor, University of Connecticut*

EURO Athens
2021 Greece

Process Systems and Operations Research Laboratory

Background

- Food security challenges worsening due to population growth
- Alternative approaches needed to supplement conventional food production
- Controlled environment agriculture (CEA)
  - Advantages
    - Sustainability
    - Increased food security
  - Disadvantages
    - High startup costs
    - Venture risk

Motivation

General-use CEA design and planning model needed to economically motivate adoption of CEA technology.

Traditional agricultural planning strategies are…

- Reactive, system-specific, focused on yield-based uncertainty

revenue = yield x market price

Our proposed CEA design/planning strategy is…

- Proactive, general, focused on market price uncertainty

Objective

To simultaneously optimize the design and scheduling of CEA systems for robustness to market uncertainty.

Model Features:

General

- One model for economically optimal engineering design of CEA systems

Flexible

- Demand-based constraints, nutritional constraints, non-constant grow periods, capital and operating cost models, location cost models

Efficient

- Exploit existing solvers (IPOPT, Gurobi, EAGO) to rapidly assess economic viability of CEA systems with minimal modification required

Trader's Perspective

- Modern portfolio theory and robust optimization applied to agriculture
- Minimize portfolio risk under worst-case uncertainty
- Portfolios robust to user-specified risk level

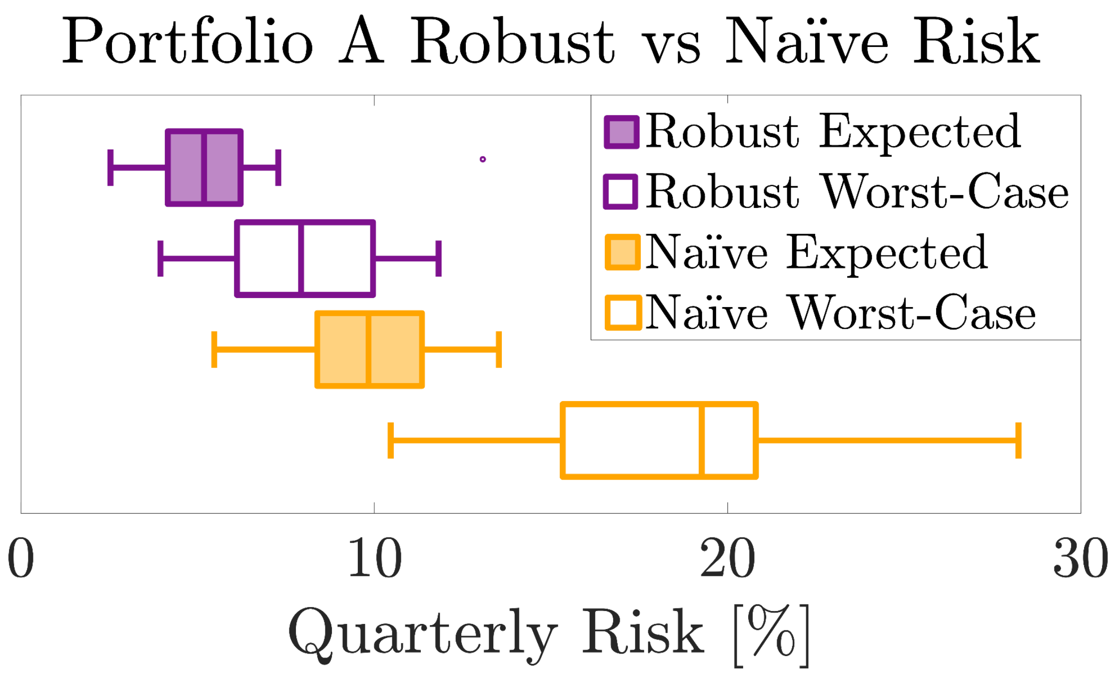
Min-Max Formulation

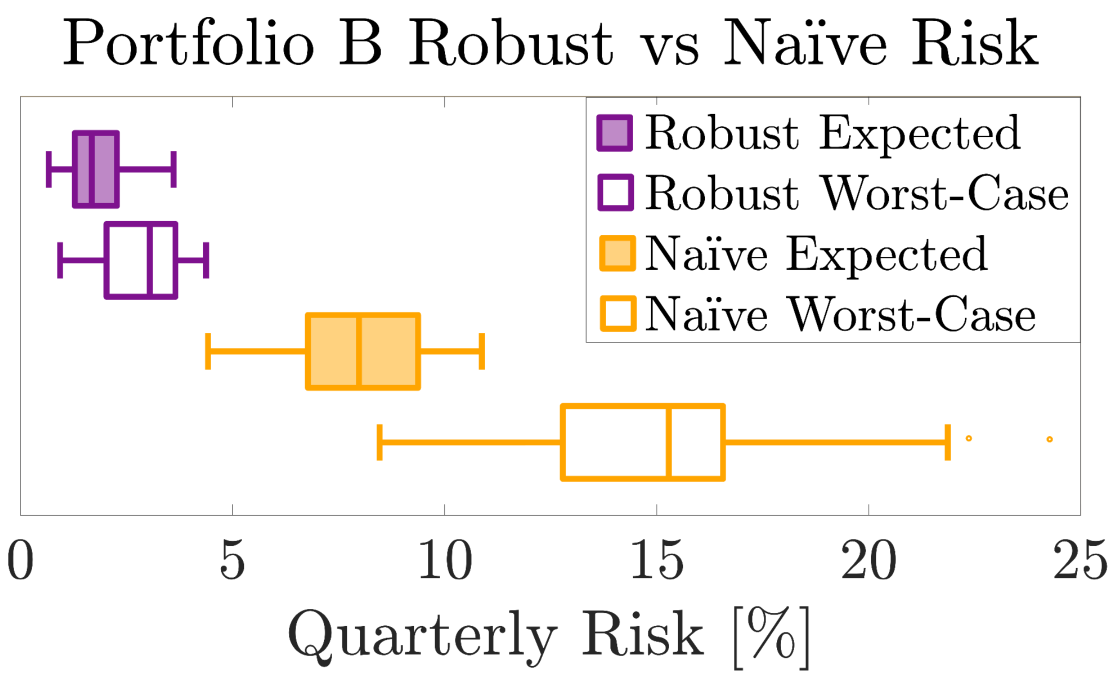
$$
\begin{aligned}
f^* &= \min_{\mathbf{x}\in\mathcal{X}} \max_{\mathbf{M}\in\mathcal{M}} \mathbf{x}^{\mathrm{T}}\mathbf{M}\mathbf{x} - t_r \\
\text{s.t. } & \mathbf{r}^{\mathrm{T}}\mathbf{x} \geq r_{\min} \\
& \mathbf{1}^{\mathrm{T}}\mathbf{x} = 1 \\
& \mathbf{M} \succeq 0 \\
& \mathcal{M} \in \mathbb{R}^{n_c \times n_c},\ \mathcal{X} = [0,1]^{n_c}
\end{aligned}
$$

SIP Reformulation

$$
\begin{aligned}
\eta^* &= \min_{\eta\in H\subset\mathbb{R},\mathbf{x}\in\mathcal{X}} \eta \\
\text{s.t. } & \mathbf{r}^{\mathrm{T}}\mathbf{x} \geq r_{\min} \\
& \mathbf{1}^{\mathrm{T}}\mathbf{x} = 1 \\
& \mathbf{M} \succeq 0 \\
& \mathbf{x}^{\mathrm{T}}\mathbf{M}\mathbf{x} - t_r - \eta \leq 0,\ \forall\, \mathbf{M} \in \mathcal{M}
\end{aligned}
$$

Trader's Results



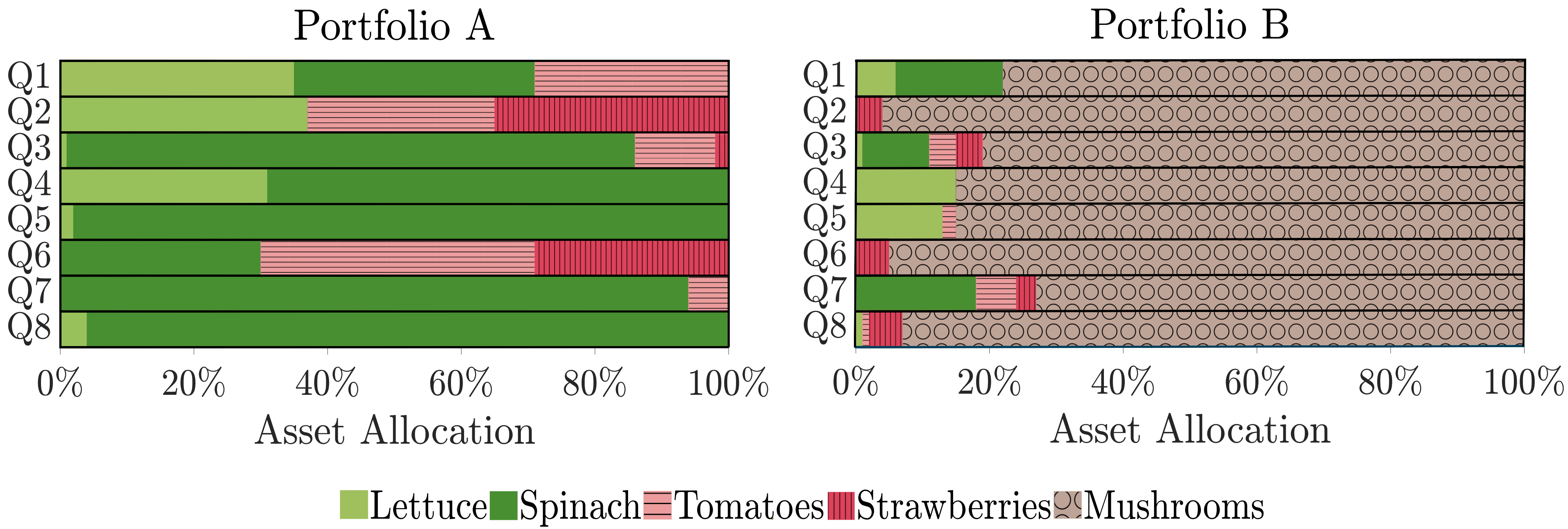


- Robust portfolios show significant risk reduction over naïve portfolios
- Diversification is an effective strategy for mitigating economic uncertainty in investment portfolios of crop-specific commodities

Grower’s Perspective

Simultaneous optimization of system design and operation.

- **d** Design variables: capacity, location, spatial allocation of each grow mode
- **X** Scheduling variables: crop allocations for each grow period



Grower's Model SIP

$$f^* = \max_{\mathbf{d}\in D, \mathbf{X}\in\Xi} f_{\mathrm{NPV}}(\mathbf{d}, \mathbf{X})$$

$$\text{s.t. } \mathbf{1}^{\mathrm{T}}\mathbf{x}_j = 1,\ j = 1,\dots,n_p$$

$$d_{z+2} = \left(\sum_{\zeta=1}^{\mu} d_{\zeta+2}\right)\left(\sum_{i\in\kappa_z} x_{ij}\right),\ j = 1,\dots,n_p,\ z = 1,\dots,\mu$$

$$\mathbf{Qp}_{\min} \le \left(\sum_{\zeta=1}^{\mu} d_{\zeta+2}\right)\left(\mathbf{Y}\sum_{j=1}^{4}\mathbf{x}_{j+4(q-1)}\right) \le \mathbf{Qp}_{\max},\ q = 1,\dots,n_y$$

$$\mathbf{x}_j^{\mathrm{T}}\mathbf{M}_j\mathbf{x}_j - t_r \le 0,\ \ \forall\, \mathbf{M}_j \in M_j \in \mathbb{R}^{n_c\times n_c},\ j = 1,\dots,n_p$$

$$\mathbf{M}_j \succeq 0,\ j = 1,\dots,n_p$$

Nonconvex objective function

- Maximize NPV over project lifespan
- Capital and operating expenses
- Annual cash-flow discounting

Nonconvex Objective

$$f_{NPV}(\mathbf{d}, \mathbf{X}) = C_{rev}(\mathbf{d}, \mathbf{X}) - (1 + P)\, C_{cap}(\mathbf{d}) - C_{op}(\mathbf{d}, \mathbf{X})$$

$C_{rev}(\mathbf{d}, \mathbf{X})$ – revenue as a function of design and allocations
$C_{cap}(\mathbf{d})$ – capital expenses as a function of design
$C_{op}(\mathbf{d}, \mathbf{X})$ – operating expenses as a function of design and allocations
P – capital financing coefficient

Bilinear terms, summation of convex and concave terms, and nonconvex land-cost model.

Grower's Model SIP

$$f^* = \max_{\mathbf{d}\in D, \mathbf{X}\in \Xi} f_{\mathrm{NPV}}(\mathbf{d}, \mathbf{X})$$

$$\text{s.t. } \boxed{\mathbf{1}^{\mathrm{T}}\mathbf{x}_j = 1,\ j = 1,\ldots,n_p}$$

$$d_{z+2} = \left(\sum_{\zeta=1}^{\mu} d_{\zeta+2}\right)\left(\sum_{i\in\kappa_z} x_{ij}\right),\ j = 1,\ldots,n_p,\ z = 1,\ldots,\mu$$

$$\mathbf{Qp}_{\min} \le \left(\sum_{\zeta=1}^{\mu} d_{\zeta+2}\right)\left(\mathbf{Y}\sum_{j=1}^{4} \mathbf{x}_{j+4(q-1)}\right) \le \mathbf{Qp}_{\max},\ q = 1,\ldots,n_y$$

$$\mathbf{x}_j^{\mathrm{T}}\mathbf{M}_j\mathbf{x}_j - t_r \le 0,\ \ \forall\, \mathbf{M}_j \in M_j \in \mathbb{R}^{n_c\times n_c},\ j = 1,\ldots,n_p$$

$$\mathbf{M}_j \succeq 0,\ j = 1,\ldots,n_p$$

Nonconvex objective function

Crop allocation constant sum

Grower's Model SIP

$$f^* = \max_{\mathbf{d}\in D, \mathbf{X}\in\Xi} f_{\mathrm{NPV}}(\mathbf{d}, \mathbf{X})$$

Nonconvex objective function

$$\text{s.t. } \mathbf{1}^{\mathrm{T}}\mathbf{x}_j = 1,\ j = 1,\ldots,n_p$$

Crop allocation constant sum

$$d_{z+2} = \left(\sum_{\zeta=1}^{\mu} d_{\zeta+2}\right)\left(\sum_{i\in\kappa_z} x_{ij}\right),\ j = 1,\ldots,n_p,\ z = 1,\ldots,\mu$$

Capacity dedicated to a single grow mode

$$\mathbf{Qp}_{\min} \le \left(\sum_{\zeta=1}^{\mu} d_{\zeta+2}\right)\left(\mathbf{Y}\sum_{j=1}^{4}\mathbf{x}_{j+4(q-1)}\right) \le \mathbf{Qp}_{\max},\ q = 1,\ldots,n_y$$

$$\mathbf{x}_j^{\mathrm{T}}\mathbf{M}_j\mathbf{x}_j - t_r \le 0,\ \forall\, \mathbf{M}_j \in M_j \in \mathbb{R}^{n_c\times n_c},\ j = 1,\ldots,n_p$$

$$\mathbf{M}_j \succeq 0,\ j = 1,\ldots,n_p$$

Grower's Model SIP

$$f^* = \max_{\mathbf{d}\in D, \mathbf{X}\in\Xi} f_{\text{NPV}}(\mathbf{d}, \mathbf{X})$$

Nonconvex objective function

$$\text{s.t.}\ \ \mathbf{1}^{\text{T}}\mathbf{x}_j = 1,\ j = 1,\ldots,n_p$$

Crop allocation constant sum

$$d_{z+2} = \left(\sum_{\zeta=1}^{\mu} d_{\zeta+2}\right)\left(\sum_{i\in\kappa_z} x_{ij}\right),\ j = 1,\ldots,n_p,\ z = 1,\ldots,\mu$$

Capacity dedicated to a single grow mode

$$\mathbf{Qp}_{\min} \le \left(\sum_{\zeta=1}^{\mu} d_{\zeta+2}\right)\left(\mathbf{Y}\sum_{j=1}^{4}\mathbf{x}_{j+4(q-1)}\right) \le \mathbf{Qp}_{\max},\ q = 1,\ldots,n_y$$

Production limited by demand

$$\mathbf{x}_j^{\text{T}}\mathbf{M}_j\mathbf{x}_j - t_r \le 0,\ \ \forall\, \mathbf{M}_j \in M_j \in \mathbb{R}^{n_c\times n_c},\ j = 1,\ldots,n_p$$

$$\mathbf{M}_j \succeq 0,\ j = 1,\ldots,n_p$$

Grower's Model SIP

$$f^* = \max_{\mathbf{d}\in D, \mathbf{X}\in\Xi} f_{\mathrm{NPV}}(\mathbf{d}, \mathbf{X})$$

Nonconvex objective function

$$\text{s.t. } \mathbf{1}^{\mathrm{T}}\mathbf{x}_j = 1, \; j = 1, \ldots, n_p$$

Crop allocation constant sum

$$d_{z+2} = \left(\sum_{\zeta=1}^{\mu} d_{\zeta+2}\right)\left(\sum_{i\in\kappa_z} x_{ij}\right), \; j = 1, \ldots, n_p, \; z = 1, \ldots, \mu$$

Capacity dedicated to a single grow mode

$$\mathbf{Qp}_{\min} \le \left(\sum_{\zeta=1}^{\mu} d_{\zeta+2}\right)\left(\mathbf{Y}\sum_{j=1}^{4} \mathbf{x}_{j+4(q-1)}\right) \le \mathbf{Qp}_{\max}, \; q = 1, \ldots, n_y$$

Production limited by demand

$$\mathbf{x}_j^{\mathrm{T}}\mathbf{M}_j\mathbf{x}_j - t_r \le 0, \;\; \forall\, \mathbf{M}_j \in M_j \in \mathbb{R}^{n_c\times n_c}, \; j = 1, \ldots, n_p$$

SIP constraints controlling multi-period risk exposure

$$\mathbf{M}_j \succeq 0, \; j = 1, \ldots, n_p$$

Grower's Model SIP

$$f^* = \max_{\mathbf{d}\in D, \mathbf{X}\in\Xi} f_{\mathrm{NPV}}(\mathbf{d}, \mathbf{X})$$

Nonconvex objective function

$$\text{s.t. } \mathbf{1}^{\mathrm{T}}\mathbf{x}_j = 1,\ j = 1,\ldots,n_p$$

Crop allocation constant sum

$$d_{z+2} = \left(\sum_{\zeta=1}^{\mu} d_{\zeta+2}\right)\left(\sum_{i\in\kappa_z} x_{ij}\right),\ j = 1,\ldots,n_p,\ z = 1,\ldots,\mu$$

Capacity dedicated to a single grow mode

$$\mathbf{Qp}_{\min} \le \left(\sum_{\zeta=1}^{\mu} d_{\zeta+2}\right)\left(\mathbf{Y}\sum_{j=1}^{4} \mathbf{x}_{j+4(q-1)}\right) \le \mathbf{Qp}_{\max},\ q = 1,\ldots,n_y$$

Production limited by demand

$$\mathbf{x}_j^{\mathrm{T}}\mathbf{M}_j\mathbf{x}_j - t_r \le 0,\ \ \forall\, \mathbf{M}_j \in M_j \in \mathbb{R}^{n_c\times n_c},\ j = 1,\ldots,n_p$$

SIP constraints controlling multi-period risk exposure

$$\mathbf{M}_j \succeq 0,\ j = 1,\ldots,n_p$$

PSD covariance matrix

Grower's Model SIP

$$f^* = \max_{\mathbf{d}\in D, \mathbf{X}\in\Xi} f_{\text{NPV}}(\mathbf{d}, \mathbf{X})$$

Nonconvex objective function

$$\text{s.t. } \mathbf{1}^{\mathrm{T}}\mathbf{x}_j = 1,\ j = 1,\ldots,n_p$$

Crop allocation constant sum

$$d_{z+2} = \left(\sum_{\zeta=1}^{\mu} d_{\zeta+2}\right)\left(\sum_{i\in\kappa_z} x_{ij}\right),\ j = 1,\ldots,n_p,\ z = 1,\ldots,\mu$$

Capacity dedicated to a single grow mode

$$\mathbf{Qp}_{\min} \le \left(\sum_{\zeta=1}^{\mu} d_{\zeta+2}\right)\left(\mathbf{Y}\sum_{j=1}^{4}\mathbf{x}_{j+4(q-1)}\right) \le \mathbf{Qp}_{\max},\ q = 1,\ldots,n_y$$

Production limited by demand

$$\mathbf{x}_j^{\mathrm{T}}\mathbf{M}_j\mathbf{x}_j - t_r \le 0,\ \ \forall\, \mathbf{M}_j \in M_j \in \mathbb{R}^{n_c\times n_c},\ j = 1,\ldots,n_p$$

SIP constraints controlling multi-period risk exposure

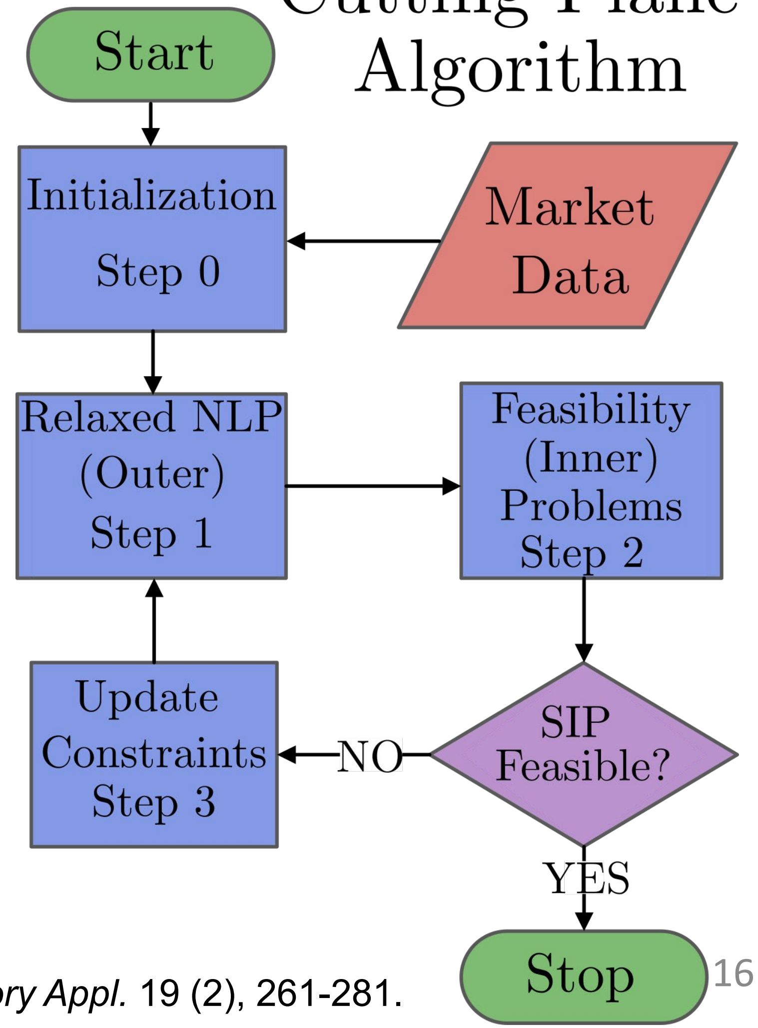
$$\mathbf{M}_j \succeq 0,\ j = 1,\ldots,n_p$$

PSD covariance matrix

Methodology: SIP

- The SIP must be solved to global optimality.
- We solve the SIP using the Blankenship & Falk cutting-plane algorithm.
- Construct SIP feasible set by iteratively solving the relaxed NLP (discretization-based procedure) and the feasibility problems.
- Use EAGO spatial B&B and JuMP for mathematical optimization in Julia

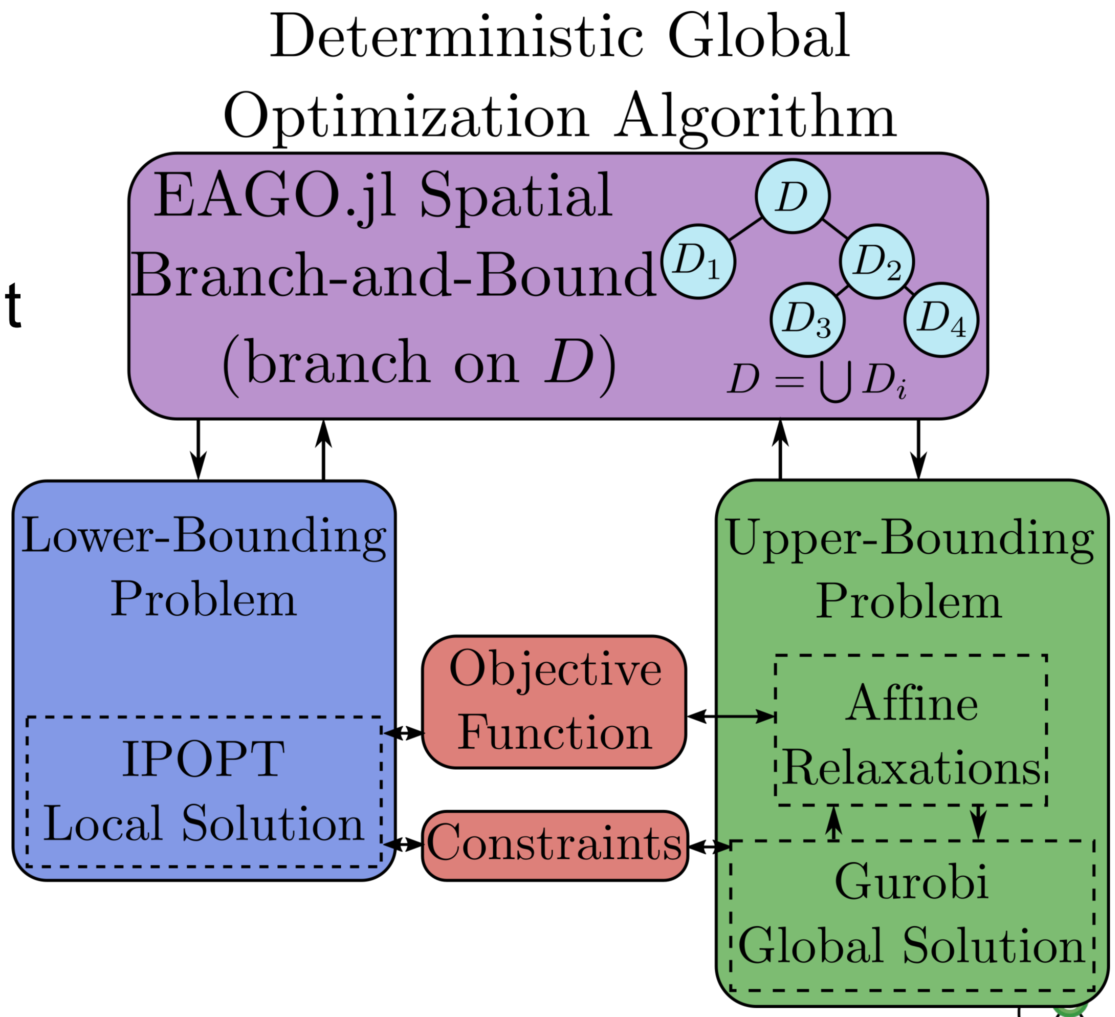
Cutting-Plane Algorithm



Blankenship, J.W., Falk, J.E., 1976. **Infinitely constrained optimization problems.** *J. Optim. Theory Appl.* 19 (2), 261-281.

Methodology: SIP Subproblems

- We employ EAGO spatial B&B with custom upper- and lower-bounding problems.
- Lower-bounding problem
  - Solve NLP locally using IPOPT at current node in B&B tree
- Upper-bounding problem
  - Partially relax NLP to obtain affine and bilinear terms only, solve to global optimality using Gurobi's nonconvex solver
- Feasibility subproblems are SDPs solved reliably using SCS.



Methodology: Upper-Bounding Problem

$$f_{NPV}(\mathbf{d},\mathbf{X}) = C_{rev}(\mathbf{d},\mathbf{X}) - (1+P)\, C_{cap}(\mathbf{d}) - C_{op}(\mathbf{d},\mathbf{X})$$

$C_{rev}(\mathbf{d},\mathbf{X})$, $C_{cap}(\mathbf{d})$, $C_{op}(\mathbf{d},\mathbf{X})$ all possess bilinear terms

Methodology: Upper-Bounding Problem

$$f_{NPV}(\mathbf{d}, \mathbf{X}) = C_{rev}(\mathbf{d}, \mathbf{X}) - (1 + P)\, \boxed{C_{cap}(\mathbf{d})} - C_{op}(\mathbf{d}, \mathbf{X})$$

$C_{rev}(\mathbf{d}, \mathbf{X})$, $C_{cap}(\mathbf{d})$, $C_{op}(\mathbf{d}, \mathbf{X})$ all possess bilinear terms

$C_{cap}(\mathbf{d})$ also contains more complicated exponential and power terms

Methodology: Upper-Bounding Problem

$$f_{NPV}(\mathbf{d},\mathbf{X}) = C_{rev}(\mathbf{d},\mathbf{X}) - (1+P)\,\boxed{C_{cap}(\mathbf{d})} - C_{op}(\mathbf{d},\mathbf{X})$$

- Employ EAGO spatial B&B to partially relax NLP by constructing affine relaxations of exponential and power terms.
- Branch on design variables only!
- Pass to Gurobi which branches on both $\mathbf{d}$ and $\mathbf{X}$ for the current node of EAGO's tree.
- Gurobi solves the partially relaxed NLP to global optimality using its nonconvex solver with bilinear relaxations.

Results - Methodology

Grower's Model Run Time

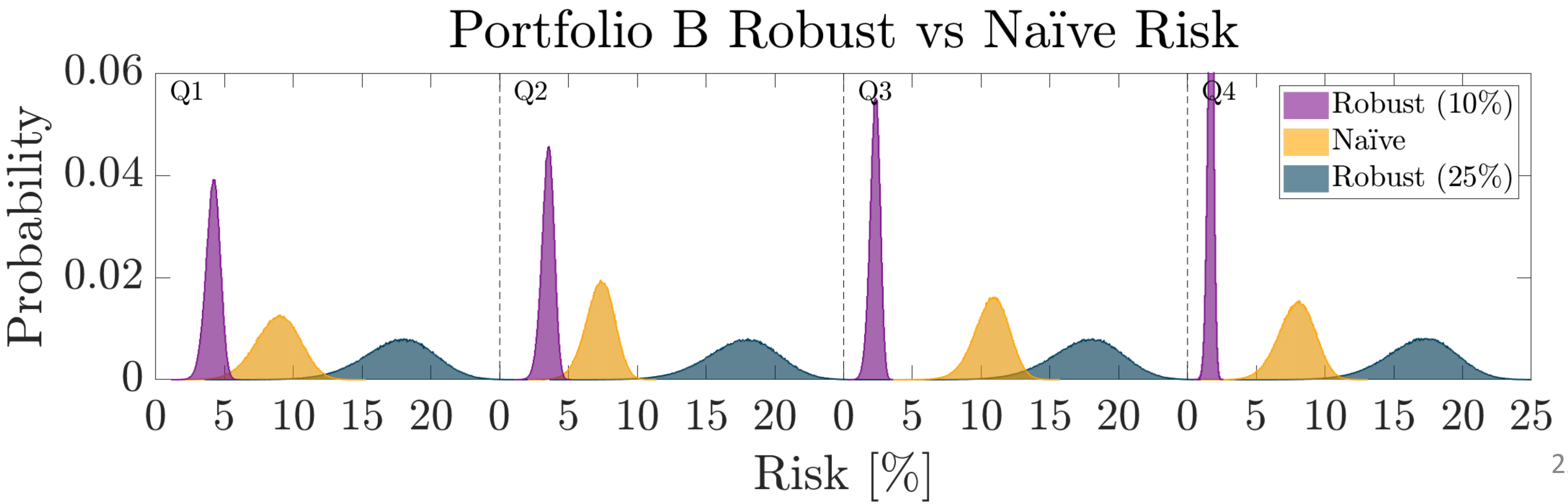
Tolerable Risk	10%	14%	21.75%	22.5%	23%	25%
Portfolio A	-	-	133	16	9.3	4.4
Portfolio B	288	379	8.9	5.4	5.2	6.2

Largest problem solved:

- 44 upper-level decision variables
- 8 semi-infinite constraints parameterized by 25-dimensional uncertainty sets
- 6-288s for 10-25% tolerable risk levels

Results - Application

- Risk under Monte-Carlo simulated market returns to assess performance of robust vs naïve approach
- Robust design achieves significant risk reduction



NPVs of Robust Systems

NPV of Robust Optimal CEA Systems

Tolerable Risk	**10%**	**14%**	**21.75%**	**22.5%**	**23%**	**25%**
Portfolio A	-	-	17.9	20.5	21.4	21.7
Portfolio B	9.5	9.5	19.6	21.2	21.3	21.3

- NPV of robust optimal design with robust optimal allocations implemented reported in million USD
- All NPVs are positive means all robust optimal solutions obtained represent an economically feasible design

Robust vs Naïve Allocations

NPV Reduction with Naïve Allocations

Tolerable Risk	10%	14%	21.75%	22.5%	23%	25%
Portfolio A	–	–	-195%	-165%	-157%	-155%
Portfolio B	-116%	-116%	-201%	-182%	-180%	-180%

- For a fixed design we observe over 100% reduction in NPV with naïve crop allocations in all simulated cases
- The economic feasibility of CEA systems is nontrivial

Summary & Conclusions

- Robust design and scheduling of CEA systems
- Developed novel SIP formulation and solution methodology
- Economically viable robust systems designs
- Foundation for higher-complexity CEA design and planning applications

Acknowledgements

Members of the process systems and operations research laboratory at the University of Connecticut (https://psor.uconn.edu/)

Prof. Matthew D. Stuber
Matthew Wilhelm

Process Systems and
Operations Research
Laboratory

UCONN
UNIVERSITY OF CONNECTICUT

EURO ATHENS 2021 GREECE HYBRID

Thank you for your attention!

Process Systems and Operations Research Laboratory